7.3 Use Functions Involving *e*

Before You studied exponential growth and decay functions.
Now You will study functions involving the natural base *e*.
Why? So you can model visibility underwater, as in Ex. 59.

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

Simplify the expression.

1. $\frac{21x^{12}y^3}{45x^5y^{-3}}$ $\frac{7x^7y^6}{15}$
2. $(-2a^3b^2)^4$ $16a^{12}b^8$
3. $\sqrt[3]{64r^9s^6t^3}$ $4r^3s^2t$
4. An account with a balance of \$1000 pays 3.65% annual interest, compounded daily. What is the balance at the end of 1 year if no money is added to the principal? **\$1037.17**

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 192–194.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with 7.4
• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1, p. 477

When is the natural base e useful?

Tell students they will learn how to answer this question by studying exponential functions related to continuous change.

NCTM STANDARDS

Standard 6: Solve problems in math and other contexts

Standard 10: Use representations to interpret phenomena

Key Vocabulary
• **natural base *e***

The history of mathematics is marked by the discovery of special numbers such as π and i. Another special number is denoted by the letter e. The number is called the **natural base *e*** or the *Euler number* after its discoverer, Leonhard Euler (1707–1783). The expression $\left(1 + \frac{1}{n}\right)^n$ approaches e as n increases.

n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

KEY CONCEPT *For Your Notebook*

The Natural Base *e*

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718281828$.

REVIEW EXPONENTS
For help with properties of exponents, see p. 330.

EXAMPLE 1 **Simplify natural base expressions**

Simplify the expression.

a. $e^2 \cdot e^5 = e^{2+5} = e^7$

b. $\frac{12e^4}{3e^3} = 4e^{4-3} = 4e$

c. $(5e^{-3x})^2 = 5^2(e^{-3x})^2 = 25e^{-6x} = \frac{25}{e^{6x}}$

EXAMPLE 2 **Evaluate natural base expressions**

Use a calculator to evaluate the expression.

	Expression	Keystrokes	Display
a.	e^4	2nd [e^x] 4) ENTER	54.59815003
b.	$e^{-0.09}$	2nd [e^x] (−) .09) ENTER	0.9139311853

Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 27–28)
- Practice levels A, B, C (pp. 30–32)
- Study Guide (pp. 33–34)
- Catch-up for Absent Students (p. 35)
- Application (p. 36)
- Challenge (p. 37)

Workbooks
- Notetaking Guide (pp. 192–194)
- Practice Workbook (pp. 112–113)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the Algebra 2 Toolkit for more strategies for meeting individual needs.

GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $e^7 \cdot e^4$ e^{11}
2. $2e^{-3} \cdot 6e^5$ $12e^2$
3. $\frac{24e^8}{4e^5}$ $6e^3$
4. $(10e^{-4x})^3$ $\frac{1000}{e^{12x}}$
5. Use a calculator to evaluate $e^{3/4}$. about 2.117

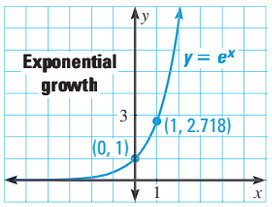
KEY CONCEPT For Your Notebook

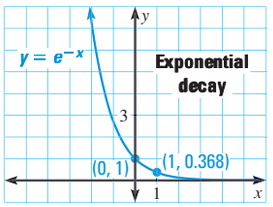
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- If $a > 0$ and $r > 0$, the function is an exponential growth function.
- If $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.





EXAMPLE 3 Graph natural base functions

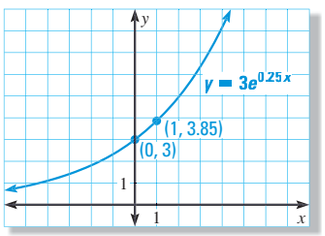
Graph the function. State the domain and range.

a. $y = 3e^{0.25x}$

b. $y = e^{-0.75(x-2)} + 1$

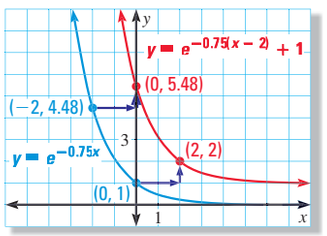
Solution

a. Because $a = 3$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Plot the points (0, 3) and (1, 3.85) and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

b. $a = 1$ is positive and $r = -0.75$ is negative, so the function is an exponential decay function. Translate the graph of $y = e^{-0.75x}$ right 2 units and up 1 unit.



The domain is all real numbers, and the range is $y > 1$.

ANOTHER WAY

You can also write the function from part (a) in the form $y = ab^x$ in order to graph it:

$y = 3e^{0.25x}$

$y = 3(e^{0.25})^x$

$y \approx 3(1.28)^x$

Differentiated Instruction

Advanced Encourage students to do Internet and/or library research on the history and significance of the number e. You could ask these students to work individually or in a small group and to either write reports on their findings or do a presentation for their classmates. Either a written report or oral presentation should include graphics.

See also the Algebra 2 Toolkit for more strategies.

Motivating the Lesson

You and your parents are saving money for your education in an account. You can use a formula with the natural base e to estimate how much money will be in your account when the money is needed.

3 TEACH

Extra Example 1

Simplify the expression.

a. $e^9 \cdot e^6$ e^{15}

b. $\frac{60e^8}{12e^3}$ $5e^5$

c. $(-10e^{-5x})^3$ $\frac{-1000}{e^{15x}}$

Extra Example 2

Use a calculator to evaluate the expression.

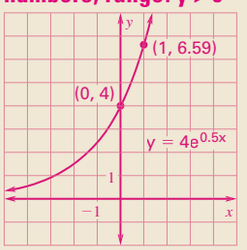
a. e^6 403.429

b. $e^{-0.28}$ 0.756

Extra Example 3

Graph the function. State the domain and range.

a. $y = 4e^{0.5x}$ domain: all real numbers; range: $y > 0$



b. $y = e^{-1.5(x+2)} - 4$ domain: all real numbers; range: $y > -4$